Statistics

Lecture 11

Feb 19-8:47 AM

Consider the chart below:

x	$P(x)$
1	.01
2	.1
3	.24
4	.3
5	.35
6	0

1) $P(x=6)$

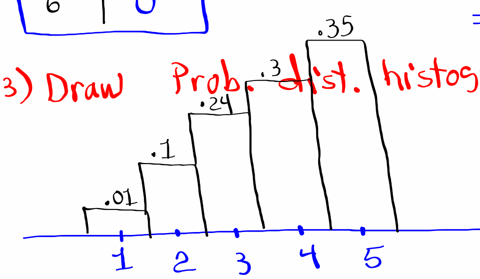
$= 1 - [.01 + .1 + .24 + .3 + .35]$

$= 1 - 1 = \boxed{0}$

2) $P(x \geq 2) = 1 - P(x=1)$

$= 1 - .01 = \boxed{.99}$

3) Draw Prob. dist. histogram



$\mu = \bar{x} = 3.88$

$\sigma = \sigma_x = 1.032$

$n = 1$

4) Find μ & σ

$x \to L1$

$P(x) \to L2$

STAT → CALC

1:1-Var Stats

List: L1, FreqList: L2

Jan 31-4:29 PM

Find σ^2 in reduced fraction

VARS | 5: Statistics | 4: σ_x | x^2 | Enter

1.0656

MATH | 1: ▶Frac | Enter

$\frac{666}{625}$

Round μ & σ to whole numbers

$\mu = 3.88$
$\sigma = 1.032$ ⇒ $\mu = 4$, $\sigma = 1$

68% Range $\mu \pm \sigma = 4 \pm 1 \rightarrow$ 3 to 5

95% Range $\mu \pm 2\sigma = 4 \pm 2(1) \rightarrow$ 2 to 6

Usual Range

99.7% Range $\mu \pm 3\sigma = 4 \pm 3(1) \rightarrow$ 1 to 7

Jan 31-4:38 PM

4000 Tkts sold for \$100 each.

one ticket is drawn, winner gets a car worth \$25000

Find expected value per ticket sold.

Net	P(Net)	
100 − 25000	1/4000	winning TKT
100 − 0	3999/4000	losing tkts

Net → x → L1

P(Net) → P(x) → L2

Calc
1-Var Stats
with L1 & L2

$E.V. = \mu = \bar{x}$

\$93.75

Fundraisers make \$93.75 / Tkt.

Jan 31-4:44 PM

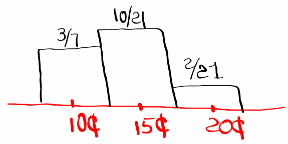
5 Dimes, 10 Nickels, Select 2 Coins, No replacement

NN → 10¢ $P(10¢) = \frac{_{10}C_2}{_{15}C_2} = \frac{45}{105} = \frac{3}{7}$

ND, DN → 15¢ $P(15¢) = \frac{_5C_1 \cdot {_{10}C_1}}{_{15}C_2} = \frac{50}{105} = \frac{10}{21}$

DD → 20¢ $P(20¢) = \frac{_5C_2}{_{15}C_2} = \frac{10}{105} = \frac{2}{21}$

T	P(T)
10¢	$\frac{3}{7}$
15¢	$\frac{10}{21}$
20¢	$\frac{2}{21}$



Find μ & σ

T → L1
P(T) → L2

1-Var Stats with L1 & L2

$\mu = \bar{x} = 13.\bar{3}$

$\sigma = \sigma_x = 3.212$

$n = 1$

Find σ^2 in reduced fraction → $\frac{650}{63}$

[VARS] [5: Statistics] [4: σ_x] [x^2] [MATH] [1: ▶Frac] [Enter]

68% Range

$\mu \pm \sigma = 13 \pm 3$ → 10 – 16

95% Range

$\mu \pm 2\sigma = 13 \pm 2(3)$ → 7 To 19

Jan 31-4:49 PM

Consider a binomial Prob. dist with $n = 25$ and $p = .8$ (SG 16)

1) $q = 1 - p = .2$

2) $np = 25(.8) = 20$

3) $npq = 25(.8)(.2) = 4$

4) $\sqrt{npq} = \sqrt{4} = 2$

5) Find $P(x = 18)$ = binompdf(25, .8, 18) = .111

(n, p, x; x = # of Successes; 2nd VARS ↓↓…↓)

Jan 31-5:01 PM

You are taking a True/ False exam with 100 questions. You are making random guesses.

Success is to guess correctly.

$n = 100$ $\quad p = .5$ $\quad q = .5$

$np = 100(.5) = \boxed{50}$ $\quad npq = 100(.5)(.5) = \boxed{25}$ $\quad \sqrt{npq} = \sqrt{25} = \boxed{5}$

Find P(guess exactly 45 correct answers)

$= P(x = 45) = \text{binompdf}(100, .5, 45) = \boxed{.048}$

Find P(guess at most 45 correct answers)

$= P(x \le 45) = \text{binomcdf}(100, .5, 45) = \boxed{.184}$

Find P(guess fewer than 60 correct answers)

$= P(x < 60) = P(x \le 59) = \text{binomcdf}(100, .5, 59)$

$= \boxed{.972}$

Jan 31-5:07 PM

A loaded coin is tossed 150 times.

Prob. of landing tails is .6.

Success is to land tails.

1) $n = \boxed{150}$ $\quad$ 2) $p = \boxed{.6}$ $\quad$ 3) $q = \boxed{.4}$

4) $np = 150(.6) = \boxed{90}$ $\quad$ 5) $npq = 150(.6)(.4) = \boxed{36}$ $\quad$ 6) $\sqrt{npq} = \sqrt{36} = \boxed{6}$

7) Find P(exactly 100 tails)

$= P(x = 100) = \text{binompdf}(150, .6, 100)$

$= \boxed{.017}$

8) Find P(at most 100 tails)

$= P(x \le 100) = \text{binomcdf}(150, .6, 100) = \boxed{.961}$

9) Find P(at least 100 tails)

$= P(x \ge 100) = 1 - P(x \le 99)$

↑ Total Prob.

$= 1 - \text{binomcdf}(150, .6, 99)$

$= \boxed{.056}$

Jan 31-5:18 PM

Prob. of full recovery from certain surgery is .9.

50 patients are randomly selected.

1) $n = 50$ 2) $p = .9$ 3) $q = .1$

4) $np = 50(.9) = \boxed{45}$

5) $npq = 50(.9)(.1) = \boxed{4.5}$

6) $\sqrt{npq} = \sqrt{4.5} \approx \boxed{2.121}$

7) Find P(exactly 25 have full recovery)

$= P(x = 25) = \text{binompdf}(50, .9, 25)$

$= \boxed{9.075 \times 10^{-13}}$

$= \boxed{9.1 \times 10^{-13}}$

8) Find P(fewer than 48 have full recovery)

$P(x < 48) = P(x \le 47)$

$= \text{binomcdf}(50, .9, 47)$

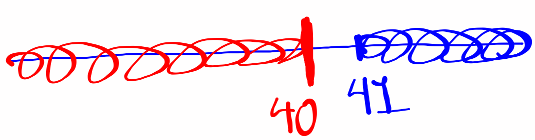
$= \boxed{.888}$

Jan 31-5:29 PM

9) Find P(more than 40 full recovery)

$P(x > 40) = P(x \ge 41)$

$= 1 - P(x \le 40)$



$= 1 - \text{binomcdf}(50, .9, 40)$

$= \boxed{.975}$

Jan 31-5:39 PM

Consider a binomial Prob. dist. with 250 independent trials and .4 prob. of Success per trial.

1) $n = 250$

2) $p = .4$

3) $q = 1 - p = .6$

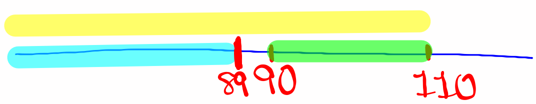
4) $np = 250(.4) = \boxed{100}$

5) $npq = 250(.4)(.6) = \boxed{60}$

6) $\sqrt{npq} = \sqrt{60} \approx \boxed{8}$

Let x be # of Successes

7) $P(90 \le x \le 110) = P(x \le 110) - P(x \le 89)$



$= \boxed{.825}$

= binomcdf(250, .4, 110) − binomcdf(250, .4, 89)

Jan 31-5:41 PM

More on Binomial Prob. dist.:

Mean μ

Variance σ^2

Standard dev. σ

$\boxed{\mu = np \quad \sigma^2 = npq \quad \sigma = \sqrt{\sigma^2}}$

100 Newborns randomly Selected.

Success is to have a girl.

$p = .5$, $q = .5$, $n = 100$

$\mu = np = 100(.5) = 50$

$\sigma^2 = npq = 100(.5)(.5) = 25$

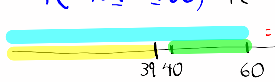
$\sigma = \sqrt{\sigma^2} = \sqrt{25} = 5$

68% Range $= \mu \pm \sigma = 50 \pm 5 \Rightarrow \boxed{45 \text{ to } 55}$

95% Range $= \mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow \boxed{40 \text{ to } 60}$

Find the Prob. of having between 40 and 60 girls, inclusive

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



= binomcdf(100, .5, 60) − binomcdf(100, .5, 39)

$= \boxed{.965}$

Jan 31-6:01 PM

You are taking a multiple-choice exam.
There are 80 questions.
Each question has 5 choices but only one correct choice.
You are making random guesses.

$n = 80$ $\quad p = \frac{1}{5} = .2$ $\quad q = \frac{4}{5} = .8$

$\mu = np = 80(.2) = \boxed{16}$ $\quad \sigma^2 = npq = 80(.2)(.8) = \boxed{12.8}$

$\sigma = \sqrt{\sigma^2} = \sqrt{12.8} = 3.578 \approx \boxed{4}$

68% Range $= \mu \pm \sigma = 16 \pm 4 \Rightarrow \boxed{12 \text{ to } 20}$

95% Range $= \mu \pm 2\sigma = 16 \pm 2(4) \Rightarrow \boxed{8 \text{ to } 24}$

P(guess correctly from 8 to 24 correct ans).

$P(8 \le x \le 24) = \text{binomcdf}(80, .2, 24) - \text{binomcdf}(80, .2, 7) = \boxed{.983}$

Jan 31-6:11 PM

ESPN says 80% of residents of SF are 49ers fan.
I randomly selected 400 residents in SF.

1) $n = 400$ $\quad$ 2) $p = .8$ $\quad$ 3) $q = .2$

4) $\mu = np = 400(.8) = \boxed{320}$ $\quad$ 5) $\sigma^2 = npq = 400(.8)(.2) = \boxed{64}$ $\quad$ 6) $\sigma = \sqrt{\sigma^2} = \sqrt{64} = \boxed{8}$

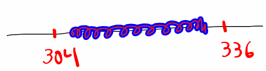
7) Usual Range
95% Range $\quad \mu \pm 2\sigma = 320 \pm 2(8) = 320 \pm 16 = \boxed{304 \text{ to } 336}$

8) Find P(between 304 and 336, inclusive, are 49er's fan.

$P(304 \le x \le 336) = \text{binomcdf}(400, .8, 336) - \text{binomcdf}(400, .8, 303) = \boxed{.961}$

If not inclusive

$P(304 < x < 336) = P(305 \le x \le 335)$



Jan 31-6:19 PM

Consider a binomial Prob. dist with $n = 150$ and $P = .4$.

P(at most two-thirds success) $\frac{2}{3} \cdot 150 = 100$

$= P(x \le 100) = \text{binomcdf}(150, .4, 100)$

$\approx \boxed{1}$ Sure event.

P(at least three-fifths success) $\frac{3}{5} \cdot 150 = 90$

$= P(x \ge 90) = 1 - P(x \le 89)$

89 90

$= 1 - \text{binomcdf}(150, .4, 89)$

$= \boxed{6.1 \times 10^{-7}}$

P(# of Successes is between 50 and 70, inclusive)
(# of Successes = x)

$P(50 \le x \le 70) = \text{binomcdf}(150, .4, 70) - \text{binomcdf}(150, .4, 49) = \boxed{.920}$

Jan 31-6:28 PM

Find μ, σ^2, σ.

$n = 150$

$P = .4$

$q = .6$

$\mu = np = 150(.4) = \boxed{60}$

$\sigma^2 = npq = 150(.4)(.6) = \boxed{36}$

$\sigma = \sqrt{\sigma^2} = \sqrt{36} = \boxed{6}$

99.7% Range $\mu \pm 3\sigma$

$= 60 \pm 3(6) = 60 \pm 18$

$\boxed{42 \text{ To } 78}$

SG 16
On Page 4

Use exact value for P & q

Jan 31-6:36 PM

CA Lotto

Pick 5 numbers from 1 to 50 in any order.

P(exactly 3 winning numbers)

$$= \frac{{}_5C_3 \cdot {}_{45}C_2}{{}_{50}C_5} = \boxed{.005}$$

P(exactly 2 winning numbers)

$$= \frac{{}_5C_2 \cdot {}_{45}C_3}{{}_{50}C_5} = \boxed{.067}$$

P(exactly 1 winning number)

$$= \frac{{}_5C_1 \cdot {}_{45}C_4}{{}_{50}C_5} = \boxed{.352}$$

Jan 31-6:41 PM

From a full standard deck of playing cards, Draw 3 cards, no replacement

P(at least 1 ace) = 1 − P(No Ace)

$$= 1 - \frac{{}_{48}C_3}{{}_{52}C_3} = \frac{1201}{5525} = \boxed{.217}$$

P(at least 1 Red Card)

$$= 1 - P(\text{No Red}) = 1 - \frac{{}_{26}C_3}{{}_{52}C_3} = \boxed{\frac{15}{17}} = \boxed{.882}$$

Jan 31-6:49 PM

x	y
3	8
4	10
4	12
5	15
6	15

SG 9

Review SG 9

Find

$a = 1$

$b = 2.5$

$r^2 = .855 \rightarrow$ 86%

$r = .925$

r is close to 1

Linear Regression is Significant.

$x \rightarrow L1$

$y \rightarrow L2$

STAT CALC

8:LinReg(a+bx)

Jan 31-6:54 PM